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Symmetric and asymmetric bound states for the nonlinear Schrödinger equation with inhomogeneous nonlinearity

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Abstract
We introduce a model of a Bose–Einstein condensate based on the one-dimensional nonlinear Schrödinger equation, in which the nonlinear term depends on the domain. The nonlinear term changes a cubic term into a quintic term, according to the domain considered. We study the existence, stability and bifurcation of solutions, and use the qualitative theory of dynamical systems to study certain properties of such solutions.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

Interest in the Bose–Einstein condensates (BEC) has increased in recent years. In this context, many types of nonlinear structures have been predicted to exist or have been experimentally observed, among which we can cite dark [1] and bright [2] solitons, gap solitons [3], etc.

From a theoretical standpoint, and for experimentally relevant conditions, the static and dynamical properties of a BEC can be described by means of an effective mean-field equation known as the Gross–Pitaevskii (GP) equation [4, 5]. This is a variant of the famous nonlinear Schrödinger (NLS) equation, which is known to be a universal model describing the evolution of complex field envelopes in nonlinear dispersive media [6]. The relevance and importance of the NLS model is not limited to the cases of conservative systems, theory of solitons and exact solutions [6–8]; in fact, the NLS equation is directly connected to dissipative universal models, such as the complex Ginzburg–Landau equation [9].

In BEC applications, the possibility of using Feschbach resonances to control the nonlinearities [10, 11] has led to the proposal of many different nonlinear phenomena induced by the manipulation of the scattering length in time [12–14], in space [15, 16] or both [17].

A new phenomenon arises when the nonlinearity changes with regard to the domain considered. There is currently a great interest in the literature dealing with the issue of spatially modulated nonlinearity [18–20], but very few papers deal with problems in which the nonlinearity changes with regard to the domain considered. We know only of the following references: in [21], the authors introduced a dynamical model of a Bose–Einstein condensate based on the two-dimensional Gross–Pitaevskii equation, in which the nonlinear coefficient is a function of radius. Within the nonlinear optic we find [22], in which the author deals with nonlinear surface waves in a certain model layered structure consisting of a layer of thickness 2D and dielectric constant ϵ, placed between two layers of a medium with a dielectric constant of the form

$$\epsilon = \epsilon + \alpha|E|^2. \tag{1}$$

A further approach to this same problem can be found in [23].

In this paper, we study the solutions of the Gross–Pitaevskii equation in a domain medium such that the internal domain with thickness $\varepsilon > 0$ has a nonlinear self-focusing quintic response, while the external domain has a self-focusing cubic response. Thus, we obtain a series of domains in a BEC, where the interactions between particles of the condensate are modeled, alternatively, for $|x| > \varepsilon$, by the cubic nonlinear Schrödinger equation,

$$-u''(x) + \omega^2 u(x) - u^3(x) = 0 \tag{2}$$

and, for $|x| < \varepsilon$, by the quintic nonlinear Schrödinger equation

$$-u''(x) + \omega^2 u(x) - u^5(x) = 0. \tag{3}$$

We should note that this model can also be introduced in nonlinear optics. Under some hypotheses, this equation could model the propagation of electromagnetic waves through a medium consisting of layers of dielectric material (see, for example, [24], for a detailed explanation of the physical background).

The remainder of this paper is organized as follows. In sections 2 and 3, we formulate the model, and present the general framework. Section 4 is devoted to the existence and bifurcation of solutions. In section 5, we use a qualitative analysis to study the solutions obtained in the previous section. We show the existence of symmetric and asymmetric solutions. Finally, section 6 contains a study of the stability of the solutions.

2. General framework

In this section, we consider a domain medium, such that the internal domain with thickness $\varepsilon > 0$ has a nonlinear self-focusing quintic response, while the external domain has a self-focusing cubic response. This model leads us to study the following differential equation:

$$-u''(x) + \omega^2 u(x) = f_\varepsilon(x, u)u(x), \qquad u \in H^1(\mathbb{R}) \tag{4}$$

where ω is the eigenvalue of the linear problem and

$$f_\varepsilon(x, u) = \begin{cases} \alpha u^4, & \text{if } \quad |\mathrm{x}| < \varepsilon \\ u^2, & \text{if } \quad |\mathrm{x}| > \varepsilon \end{cases} \tag{5}$$

with $\alpha \in \mathbb{R}$. If we introduce the characteristic function χ of the interval $[-1, 1]$, i.e.

$$\chi\left(\frac{x}{\varepsilon}\right) = \begin{cases} 1, & \text{if } \quad |\mathrm{x}| < \varepsilon \\ 0, & \text{if } \quad |\mathrm{x}| > \varepsilon \end{cases} \tag{6}$$

we can write $f_\epsilon(x, u)$ as

$$f_\varepsilon(x, u) = \alpha\chi\left(\frac{x}{\varepsilon}\right)u^4 + \left(1 - \chi\left(\frac{x}{\varepsilon}\right)\right)u^2, \tag{7}$$

and equation (4) becomes

$$-u''(x) + \omega^2 u(x) = u^3 + \chi\left(\frac{x}{\varepsilon}\right)(\alpha u^5 - u^3), \qquad u \in H^1(\mathbb{R}). \tag{8}$$

We can therefore treat this problem as a perturbative problem (see, for example, [25, 26] for an introduction to perturbative methods).

The solutions of equation (4) are the critical points on $H^1(\mathbb{R})$ of

$$I_{\varepsilon,\omega} = \frac{1}{2}\int_{\mathbb{R}} |u'|^2\,\mathrm{d}x + \frac{1}{2}\omega^2\int_{\mathbb{R}} |u|^2\,\mathrm{d}x - \frac{1}{4}\int_{\mathbb{R}} u^4\,\mathrm{d}x + G(\varepsilon, u), \tag{9}$$

where

$$G(\varepsilon, u) = \begin{cases} -\displaystyle\int_{\mathbb{R}} \left[\frac{\alpha}{6}u^6(x) - \frac{1}{4}u^4(x)\right]\chi\left(\frac{x}{\varepsilon}\right)\mathrm{d}x & \text{if} \quad \varepsilon > 0 \\ 0 & \text{if} \quad \varepsilon = 0. \end{cases} \tag{10}$$

Thus, we can write the perturbed functional $I_{\varepsilon,\omega}$ as

$$I_{\varepsilon,\omega} = I_0 + G(\varepsilon, u) \tag{11}$$

where

$$I_0 = \frac{1}{2}\int_{\mathbb{R}} |u'|^2\,\mathrm{d}x + \frac{1}{2}\,\omega^2\int_{\mathbb{R}} |u|^2\,\mathrm{d}x - \frac{1}{4}\int_{\mathbb{R}} u^4\,\mathrm{d}x \tag{12}$$

is the unperturbed functional, $I_0 \in C^2(H^1(\mathbb{R}), \mathbb{R})$, and $G \in C^3(H^1(\mathbb{R}))$ is the perturbation.

3. The unperturbed and perturbed problem

The unperturbed problem $I_0'(u) = 0$ is the equation

$$-u''(x) + \omega^2 u(x) = u^3, \qquad u \in H^1(\mathbb{R}) \tag{13}$$

which, fixed ω^2, has a unique even positive solution $u_0(x)$ such that

$$u_0(x) = \frac{\sqrt{2}\omega}{\cosh(\omega x)} \tag{14}$$

$$u_0'(0) = 0, \qquad \lim_{|x|\to\infty} u_0(x) = 0. \tag{15}$$

Since (13) is translation invariant, it follows that any

$$u_\omega(x) := u_0(x + \xi), \qquad \xi \in \mathbb{R}, \tag{16}$$

is also a solution of equation (13). Then I_0 has a (non-compact) one-dimensional critical manifold given by

$$Z = \{u_\omega(x) := u_0(x + \xi) : \xi \in \mathbb{R}\}. \tag{17}$$

It is possible to prove that Z is a non-degenerate manifold [26], i.e.,

$$Ker[I_0''(u_\xi)] = T_{u_\xi} Z, \qquad \forall\, u_\xi \in Z. \tag{18}$$

Let us define the following conditions on G, which will be useful for the proof of existence:

(i) $G \in C(\mathbb{R} \times H^1(\mathbb{R}), \mathbb{R})$ and is such that $G(0, u) = 0$, for all $u \in H^1(\mathbb{R})$. Moreover, the map $u \to G(\varepsilon, u)$ is of class C^2, for all $\varepsilon \in \mathbb{R}$, and $D_u G(\varepsilon, u)$ as well as $D_{uu} G(\varepsilon, u)$ are continuous.

(ii) There exists $\beta > 0$ such that $\|D_u G(\varepsilon, u)\|_{H^1} = o(\varepsilon^\beta)$, as $\varepsilon \to 0$.

(iii) There exists $\gamma > 0$ and $\Sigma : Z \to \mathbb{R}$ such that

$$\lim_{\varepsilon\to 0} \frac{\Sigma(\varepsilon, u)}{\varepsilon^\gamma} = \Sigma(u). \tag{19}$$

4. Existence and bifurcation of solutions

The following theorem gives the existence of solutions of equation (4) (see [25]).

Theorem 1. *Let us suppose that $I_0 \in C^2(H^1(\mathbb{R}), \mathbb{R})$ has a smooth critical manifold Z which is non-degenerate. Let G satisfy (1), (2) and (3) and let $\overline{u} \in Z$ be a strict local maximum or minimum of Σ.*

Then for $|\varepsilon|$ small the functional $I_\varepsilon = I_0 + G(\varepsilon, \cdot)$ has a critical point u_ε and if $\overline{u}$ is isolated, then $u_\varepsilon \to \overline{u}$ as $\varepsilon \to 0$.

It is clear that the function G satisfies the condition (1). We shall now prove condition (2). Through the change of variable $y = x/\varepsilon$ we find

$$|G(\varepsilon, u)| = |\varepsilon| \int_{\mathbb{R}} \left[\frac{\alpha}{6} u^6(\varepsilon y) - \frac{1}{4} u^4(\varepsilon y)\right] \chi(y)\, \mathrm{d}y. \tag{20}$$

As regards $D_u G(\varepsilon, u)$ we find, for any $\phi \in H^1(\mathbb{R})$,

$$|(D_u G(\varepsilon, u)|\phi)| = |\int_{\mathbb{R}} \varepsilon[\alpha u^5(\varepsilon y) - u^3(\varepsilon y)]\phi(\varepsilon y)\chi(y)\, \mathrm{d}y. \tag{21}$$

Since $H^1(\mathbb{R}) \subset C_B(\mathbb{R})$ we infer that

$$\begin{aligned} |(D_u G(\varepsilon, u)|\phi)| \leqslant{}& |\varepsilon| \|u\|^5_{L^\infty(\mathbb{R})} \|\phi\|_{L^\infty(\mathbb{R})} \alpha \int_{\mathbb{R}} \chi(y)\, \mathrm{d}y \\ & - |\varepsilon| \|u\|^3_{L^\infty(\mathbb{R})} \|\phi\|_{L^\infty(\mathbb{R})} \int_{\mathbb{R}} \chi(y)\, \mathrm{d}y \end{aligned} \tag{22}$$

and hence

$$\|D_u G(\varepsilon, u)\| \leqslant |\varepsilon| C \int_{\mathbb{R}} \chi(y)\, \mathrm{d}y \tag{23}$$

for some $C > 0$, proving that

$$\|D_u G(\varepsilon, u)\| \to 0 \tag{24}$$

as $\varepsilon \to 0$.

Moreover, we obtain that for $\varepsilon > 0$

$$\begin{aligned} G(\varepsilon, u_\omega(\cdot + \xi)) &= -\int_{\mathbb{R}} \chi\left(\frac{x}{\varepsilon}\right) \left[\frac{\alpha}{6} u_\omega^6(x+\xi) - \frac{1}{4} u_\omega^4(x+\xi)\right] \mathrm{d}x \\ &= -\int_{-\varepsilon}^{\varepsilon} \frac{\alpha}{6} u_\omega^6(x+\xi) - \frac{1}{4} u_\omega^4(x+\xi)\, \mathrm{d}x \\ &= -\varepsilon \int_{-1}^{1} \frac{\alpha}{6} u_\omega^6(\varepsilon x+\xi) - \frac{1}{4} u_\omega^4(\varepsilon x+\xi)\, \mathrm{d}x. \end{aligned}$$

We thus discover that the condition (3) holds with $\gamma = 1$ and

$$\Sigma_\omega(\xi) = -\frac{\alpha}{3} u_\omega^6(\xi) + \frac{1}{2} u_\omega^4(\xi). \tag{25}$$

The behavior of Σ_ω depends on the value of ω. In particular, there exists $\omega_0 = 1/\sqrt{2\alpha}$ such that, for $\varepsilon > 0$ small,

(i) For $0 < \omega < \omega_0$, Σ_ω has a unique global maximum at $\xi = 0$ (see figure 1(*a*)).
(ii) For $\omega > \omega_0$, Σ_ω has minimum at $\xi = 0$, while the global maximum is achieved at some $\pm\xi_\omega \neq 0$ (see figure 1(*b*)).

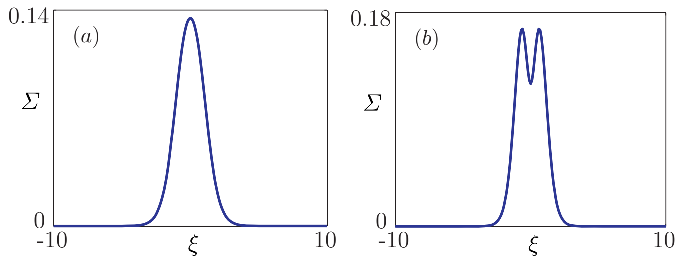

Figure 1. (a) Graph of Σ_ω, for $\omega < \omega_0$. (b) Graph of Σ_ω, for $\omega > \omega_0$. In both cases, $\alpha = 1$.

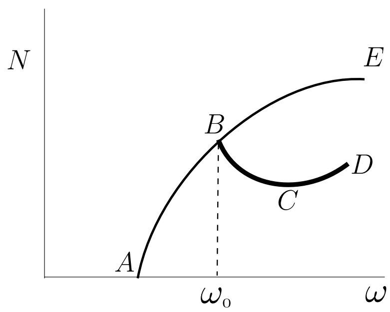

Figure 2. Bifurcation diagram for equation (4), where N is the L^2-norm of the solutions of equation (4). The curve in bold type represents the asymmetric solutions.

Thus, by using theorem 1, equation (4) has, for ε small, a solution $u_{\varepsilon,\omega}$, for all $\omega > 0$ branching from the trivial solution at $\omega = 0$. This solution corresponds to the curve AB in figure 2, which is a symmetric solution with regard to x, since $\xi = 0$. In addition, at $\omega = \omega_0$ there is a secondary bifurcation of solutions $\tilde{u}_{\varepsilon,\omega}$ of equation (4), corresponding to ξ_ω (see figure 2). Thus, the curve BE corresponds to a symmetric solution while BCD is a double curve corresponding to two asymmetric solutions, $\tilde{u}_{1\varepsilon,\omega}$ and $\tilde{u}_{2\varepsilon,\omega}$. These solutions $\tilde{u}_{i\varepsilon,\omega}$, $i = 1, 2$ are not symmetric with regard to x because $\xi_\omega \neq 0$ (see figure 1(b)).

5. Qualitative analysis and phase portraits

To corroborate the results of the previous section, we calculate the solutions of equation (4) in a qualitative manner. Thus, the solutions of equation (4) may be represented by composite phase portraits constructed from superpositions of the phase portraits of the following individual problems:

$$u' = v, \qquad v' = \omega^2 u - \alpha u^5, \qquad -\varepsilon \leqslant x \leqslant \varepsilon \tag{26}$$

and

$$u' = v, \qquad v\prime = \omega^2 u - u^3, \qquad |x| > \varepsilon. \tag{27}$$

In figure 3, we depict phase portraits for the above equations at fixed values of ω^2 and α. These phase portraits arise from a superposition of two symmetrically placed homoclinic orbits, which correspond to the cubic nonlinear Schrödinger equation, and another two symmetrically placed homoclinic orbits, which correspond to the quintic nonlinear Schrödinger equation.

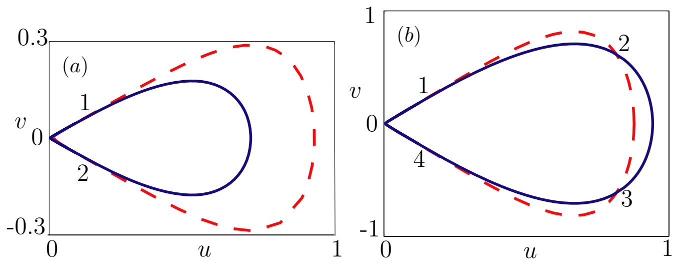

Figure 3. Composite phase portrait representing the solutions of equation (4) constructed from superpositions of two homoclinic orbits, for $\alpha = 1$. (*a*) There exists a unique symmetric wave before bifurcation ($\omega_0 = 1/\sqrt{2}$), which is given by the curve $(0-1-2-0)$ (see the text). (*b*) Beyond ω_0 a symmetric wave and two asymmetric waves can be constructed. For $\omega = 1$, we show the symmetric wave, which is the curve $0-2-3-0$, and the asymmetric waves, which are $0-2-4-0$ and $0-1-3-0$ (see the text). Both asymmetric waves are symmetric in relation to each other.

Below critical value ($\omega_0 = 1/\sqrt{2\alpha}$) a symmetric solution exists and is given by the curve $(0-1-2-0)$, where the path $0-1$ is given by the dashed red curve, the path $1-2$ is given by the solid blue curve and the path $2-0$ is again given by the dashed red curve, (see figure 3(*a*)), while beyond ω_0 more than one composite phase portrait can be constructed (figures 3(*b*)). Thus, we find a pair of asymmetric standing waves, the first of which is the curve $(0-2-4-0)$, (the path $0-2$ is given by the dashed red curve, the path $2-4$ is given by the solid blue curve and the path $4-0$ is again given by the dashed red curve) and the second of which is the curve $(0-1-3-0)$, (the path $0-1$ is given by the dashed red curve, the path $1-3$ is given by the solid blue curve and the path $3-0$ is again given by the dashed red line), together with the symmetric solution, the curve $(0-2-3-0)$, (the path $0-2$ is given by the dashed red curve, the path $2-3$ is given by the solid blue curve and the path $3-0$ is again given by the dashed red line). So, after we pass the value ω_0, we have three solutions, one symmetric and two asymmetric.

6. Remarks on stability of solutions

Here we shall briefly discuss the orbital stability of solitary waves $e^{i\omega^2 t}u_\varepsilon(x)$ corresponding to the solutions found in theorem 1.

We say that u_ε is orbitally stable if a solution $\psi(t,x)$ of the nonlinear Schrödinger equation exists for all $t \geqslant 0$ and remains H^1—close to the solitary wave $e^{i\omega^2 t}u_\varepsilon(x)$, provided $\psi(0,x)$ is sufficiently close to $u_\varepsilon(x)$ in $H^{\mathbb{R}}$. See, for example, [27, 28]. Since the orbital stability depends on ω, we will emphasize the dependence on ω by writing $u_{\varepsilon,\omega}$ instead of u_ε.

Let $m_{\varepsilon,\omega}$ denote the Morse index of $u_{\varepsilon,\omega}$ as a critical point of I_ε and let

$$\mu(\varepsilon,\omega) = \frac{\partial}{\partial\omega^2}\int_{\mathbb{R}} |u_{\varepsilon,\omega}|^2\,\mathrm{d}x. \tag{28}$$

According to [27] (see also [25]), we know that $u_{\varepsilon,\omega}$ is orbitally stable provided that $m_{\varepsilon,\omega} = 1$ and $\mu(\varepsilon,\omega) > 0$. Furthermore, if either $m_{\varepsilon,\omega} > 1$ or $m_{\varepsilon,\omega} = 1$ but $\mu(\varepsilon,\omega) < 0$, we have instability.

We can therefore evaluate the stability of the previous solutions by calculating the Morse index of the solutions $u_{\varepsilon,\omega}$ and $\tilde{u}_{\varepsilon,\omega}$. Thus, for $\omega < \omega_0$ (respectively for $\omega > \omega_0$), $u_{\varepsilon,\omega}$ corresponds to a maximum (respectively a minimum) of Σ_ω. Moreover, $\tilde{u}_{\varepsilon,\omega}(\omega > \omega_0)$

corresponds to a maximum of Σ_ω. It follows that the Morse index of $u_{\varepsilon,\omega}$ is 2 or 1, provided that, respectively, $\omega < \omega_0$ and $\omega > \omega_0$. Similarly, the Morse index of $\tilde{u}_{\varepsilon,\omega}$ is 2. Provided that the Vakhitov–Kolokolov criterion is satisfied, i.e.

$$\frac{\partial}{\partial\omega^2}\int_{\mathbb{R}} |u_{\varepsilon,\omega}|^2\,\mathrm{d}x > 0 \tag{29}$$

for $\omega > \omega_0$, one infers that the stationary wave corresponding to the symmetric solution is orbitally stable if $\omega > \omega_0$. When $\omega < \omega_0$, the stationary wave corresponding to the symmetric solution is unstable. Thus, when ω crosses ω_0 there is a change of stability: the symmetric solution becomes stable while the asymmetric solution is unstable.

7. Conclusions

In this paper, we have introduced a model of a Bose–Einstein condensate based on the one-dimensional nonlinear Schrödinger equation, in which the nonlinear term depends on the domain. We have studied the existence, stability and bifurcation of solutions, and have used the qualitative theory of dynamical systems to study some properties of such solutions.

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References

[1] Burger S, Bongs K, Dettmer S, Ertmer W, Sengstock K, Sanpera A, Shlyapnikov G V and Lewenstein M 1999 *Phys. Rev. Lett.* **83** 5198
[2] Khaykovich L, Schreck F, Ferrari G, Bourdel T, Cubizolles J, Carr L D, Castin Y and Salomon C 2002 *Science* **296** 1290
[3] Eiermann B, Anker Th, Albiez M, Taglieber M, Treutlein P, Marzlin K-P and Oberthaler M K 2004 *Phys. Rev. Lett.* **92** 230401
[4] Gross E P 1963 *J. Math. Phys.* **4** 195
[5] Pitaevskii L P 1061 *Sov. Phys.—JETP* **13** 451
[6] Sulem C and Sulem P L 1999 *The Nonlinear Schrödinger Equation* (New York: Springer)
[7] Ablowitz M J and Segur H 1981 *Solitons and The Inverse Scattering Transform* (Philadelphia: SIAM)
[8] Musslimani Z H *et al* 2008 *J. Phys. A: Math. Theor.* **41** 244019
[9] van Saarloos W and Hohenberg P C 1992 *Physica* D **56** 303
[10] Inouye S, Andrews M R, Stenger J, Miesner H J, Stamper-Kurn D M and Ketterle W 1998 *Nature* **392** 151
[11] Theis M, Thalhammer G, Winkler K, Hellwig M, Ruff G, Grimm R and Denschlag J H 2004 *Phys. Rev. Lett.* **93** 123001
[12] Abdullaev F Kh, Kamchatnov A M, Konotop V V and Brazhnyi V A 2003 *Phys. Rev. Lett.* **90** 230402
[13] Pérez-García V M, Konotop V V and Brazhnyi V A 2004 *Phys. Rev. Lett.* **92** 220403
[14] Kevrekidis P G, Pelinovsky D E and Stefanov A 2006 *J. Phys. A: Math. Gen.* **39** 479–88
[15] Belmonte-Beitia J, Pérez-García V M, Vekslerchik V and Torres P J 2007 *Phys. Rev. Lett.* **98** 064102
[16] Belmonte-Beitia J, Pérez-García V M, Vekslerchik V and Torres P J 2008 *Discrete and Continuous Dynamical Systems-Series* B **9** 221–33
[17] Belmonte-Beitia J, Pérez-García V M, Vekslerchik V and Konotop V V 2008 *Phys. Rev. Lett.* **100** 164102
[18] Sivan Y, Fibich G and Weinstein M I 2006 *Phys. Rev. Lett.* **97** 193902
[19] Sivan Y, Fibich G and Ilan B 2008 *Phys. Rev.* E **77** 045601
[20] Belmonte-Beitia J and Torres P J 2008 *J. Nonlinear Math. Phys.* **15** 65–72
[21] Sakaguchi H and Malomed B A 2006 *Phys. Rev.* E **73** 026601
[22] Akhmediev N N 1982 *Sov. Phys.—JETP* **56** 299–303

[23] Jones C K R T and Moloney J V 1986 *Phys. Lett.* A **117** 4
[24] Kivshar Y S and Agrawal G P 2003 *Optical Solitons: From Fibers to Photonic Crystals* (New York: Academic)
[25] Ambrosetti A and Malchiodi A 2006 *Perturbation Methods and Semilinear Elliptic Problems on R^n: Progress in mathematics* (Basle: Birkhäuser)
[26] Ambrosetti A, Arcoya D and Gámez J L 1998 *Rend. Sem. Mat. Univ. Padova* **100** 231–47
[27] Grillakis M, Shatah J and Strauss W 1987 *J. Funct. Anal.* **74** 160–97
Grillakis M, Shatah J and Strauss W 1990 *J. Funct. Anal.* **94** 308–48
[28] Fibich G, Sivan Y and Weinstein M 2006 *Physica* D **217**